

# Calculus Review

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Here are some practice problems to refresh your calculus I/II skills. I've only added the type of problems that will be relevant to our course. These exercises are not to be turned in, but I expect you to be able to solve them. I am aware that there are a ton of problems here and I don't expect you do them all. The point of this document is just to provide you with some practice material.

## Calculus I

### Derivatives

Calculate the derivatives of the following functions.

#### Polynomials

1.  $f(x) = 3x^2 - 4x + 1$
2.  $f(x) = 17x + 24\sqrt{x}$
3.  $f(x) = x^2 + \frac{1}{2x}$
4.  $f(x) = \sqrt{x}(x + 1)$
5.  $f(x) = -2x^{-2} + 3\sqrt{x}$
6.  $f(x) = 3x^2 + \frac{12}{\sqrt{t}} - \frac{1}{t^2}$

#### Exponentials and Trigonometric

1.  $f(x) = e^x + x^2$
2.  $f(x) = \frac{3^x}{3} + \frac{33}{\sqrt{x}}$
3.  $f(x) = e^\pi + e^x$
4.  $f(x) = \cos(x) + 2x$
5.  $f(x) = \sin(x) + 3e^x$

6.  $f(x) = \tan(x) + \sqrt{3x}$

7.  $f(x) = \tan^{-1}(x)$

**Product and Quotient Rule**

1.  $f(x) = xe^x$

2.  $f(x) = \cos(9)x\sin(x)$

3.  $f(x) = (x^2 + 3)e^x$

4.  $f(x) = (2 - x - 3x^2)(e^x - 4\sqrt{x})$

5.  $f(x) = 2xe^x - \frac{1}{\sqrt{x}}$

6.  $f(x) = \cos(x)\sin(x)x^2$

7.  $f(x) = \frac{25x^2}{e^x}$

8.  $f(x) = \frac{t+1}{t^2}$

9.  $f(x) = \frac{3x+1}{5x+1}$

10.  $f(x) = \tan(x)$  (Try to use the quotient rule rather than just stating the formula you all know and love)

11.  $f(x) = \frac{4}{\sqrt{x}}$

**Chain rule**

1.  $f(x) = (x + 1)^{2015}$

2.  $f(x) = \sqrt{1 + x^2}$

3.  $f(x) = e^{4x+1}$

4.  $f(x) = \cos(3x^2)$

5.  $f(x) = 100e^{-x^2}$

6.  $f(x) = \frac{1}{(e^x+1)^2}$

7.  $f(x) = \left(\frac{(x^2+2)}{3}\right)^2$

8.  $f(x) = xe^{x^2}$

## Optimization

In this section  $f$  will always be a differentiable function.

1. What does it mean if  $f'(a) = 0$  for some real number  $a$ ?
2. What does it mean if  $f'(a) > 0$  for some real number  $a$ ?
3. What does it mean if  $f'(a) < 0$  for some real number  $a$ ?
4. What does it mean if  $f''(a) = 0$  for some real number  $a$ ?
5. What does it mean if  $f''(a) > 0$  for some real number  $a$ ?
6. What does it mean if  $f''(a) < 0$  for some real number  $a$ ?
7. What is the difference between a local min/max and a global min/max?
8. True or False. If  $f'(a) = 0$ , then  $f$  has a local extreme value at  $a$ .
9. True or False. If  $f$  has a local extreme value at a point  $a$ , then  $f'(a) = 0$ . Does your answer change if  $f$  is not differentiable at  $a$ ?
10. Find the critical points and inflection points of the following functions:
  - (a)  $f(x) = x^3 - 9x^2 + 24x + 5$
  - (b)  $f(x) = 5x - 3\ln(x)$
  - (c)  $f(x) = 4xe^{3x}$
  - (d)  $f(x) = (x^2 - 4)^7$
  - (e)  $f(x) = x^5 + 15x^4 + 25$
11. Use the 2nd derivative test to determine the nature of the critical points you found in the previous problem. I.e. determine if they are local mins/maxs or inflection points.
12. **(IMPORTANT)** Find the absolute extreme values for the following functions subject to the constraints
  - (a)  $f(x) = x^3 - 3x^2 + 20, -1 \leq x \leq 3$
  - (b)  $f(x) = x^4 - 8x^2, -3 \leq x \leq 1$
  - (c)  $f(x) = \frac{x+1}{x^2+3}, -1 \leq x \leq 2$

## Calculus II

1.  $\int xe^{x^2} dx$
2.  $\int xe^x dx$
3.  $\int \ln(x) dx$  (Hint: Integration by parts!)
4.  $\int x\sqrt{x^2+1} dx$
5.  $\int x\sqrt{x+1} dx$

## Misc.

Be able to (roughly) graph the following functions

1.  $f(x) = (x + 2)^2$

2.  $f(x) = \ln(x)$

3.  $f(x) = e^x$

4.  $f(x) = x.$

5.  $f(x) = \cos(x)$

6.  $f(x) = \sin(x)$

7.  $f(x) = 1/x$

8.  $f(x) = |x|$

9.  $f(x) = \tan^{-1}(x)$

10.  $f(x) = x^3 - 1$